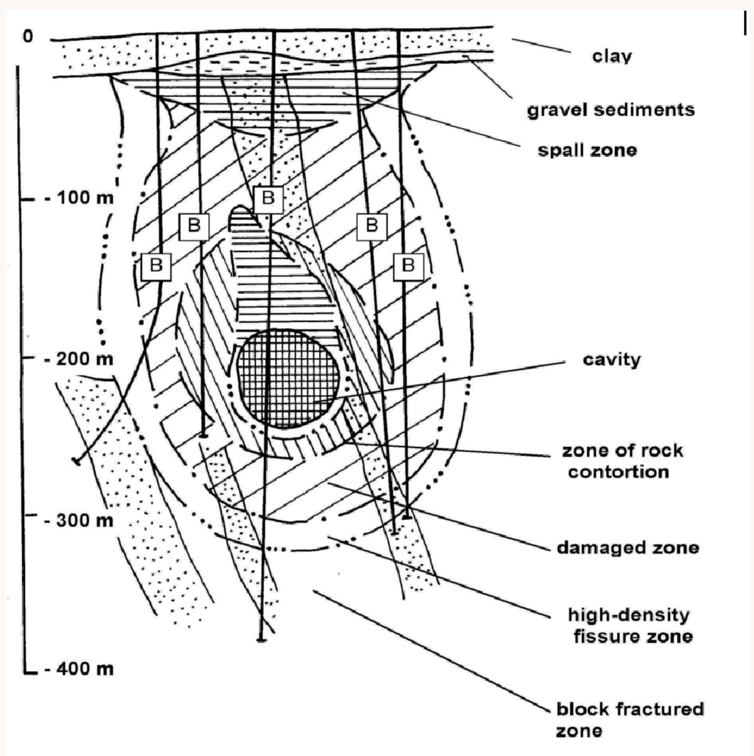
Seismic wave interaction with underground cavities

Felix Schneider [1], Sofi Esterhazy[1,2], Ilaria Perugia[2], Götz Bokelmann[1]
[1]) Department of Meteorology and Geophysics (IMGW), University of Vienna
[2]) Faculty of Mathematics, University of Vienna
felix.schneider@univie.ac.at

Motivation

The Comprehensive-Nuclear-Test-Ban-Treaty (CTBT) aim to prohibit any nuclear explosion on Earth. For its realization a nuclear test verification strategy has to be set up, consisting of three pillars:

- Monitoring
- International Data Center
- On-Site Inspection (OSI)



CTBTO PREPARATORY COMMISSION

OSI aims to be the ultimate tool to clarify wether or not a nuclear test has been carried out in violation of the CTBT. The treaty lists 17 different techniques for nuclear verification including:

→ Gamma radiation monitoring
→ Environmental sampling
→ Magnetic and Gravitational
→ Electrical conductivity measurements
→ Active and passive seismic surveys
→ Resonance seismometry

Model

In order to investigate the potential of different seismic techniques for cavity detection, we investigate the interaction of a seismic wavefield with spherical inclusions.

The simplest description of the problem is an elastic half space that contains an acoustic (gas filled) spherical cavity.

Elastic wave equation in Ω_2:

$$\rho\, \ddot{\underline{u}}_2 = (\lambda + 2\mu)\nabla(\nabla \cdot \underline{u}_2) - \mu \nabla \times \nabla \times \underline{u}_2$$

Acoustic wave equation in Ω_1:

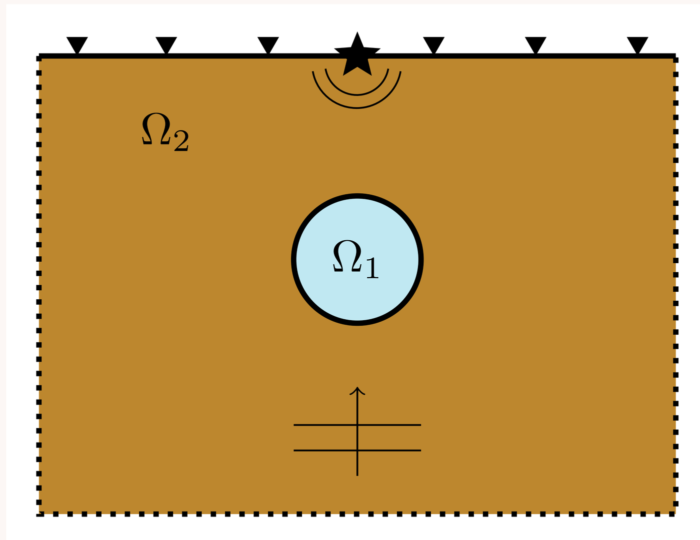
$$\rho\, \ddot{\underline{u}}_1 = \lambda\, \Delta \underline{u}_1$$

Lamé parameter:

$$\lambda = \rho(v_p^2 - 2v_s^2) \qquad \mu = \rho v_s^2$$

Boundary condition at r=R:

$$\underline{u}_2 = \underline{u}_1$$
$$\underline{\underline{\sigma}}\ \underline{n} = (\nabla \cdot \underline{u}_1)\ \underline{n}$$



Analytical approach

Set of equations for incident (U_0) and scattered fields inside$(\mathrm{U}_1)$and outside$(U_2)$of the cavity

$$\mathbf{U}_0 = \sum_{l\geq 0} \{ j_{l+1}(\omega\alpha_2 r)\mathbf{Y}_{l0}^{+} - j_{l-1}(\omega\alpha_2 r)\mathbf{Y}_{l0}^{-}\} \exp\{-i[\pi/2(l+1)]\}$$

$$\mathbf{U}_1 = \sum_{l\geq 0} \{[a_l^{(1)} j_{l+1}(\omega\alpha_1 r) + l b_l^{(1)} j_{l+1}(\omega\beta_1 r)]\mathbf{Y}_{l0}^{+} + [-a_l^{(1)} j_{l-1}(\omega\alpha_1 r) + (l+1) b_l^{(1)} j_{l-1}(\omega\beta_1 r)]\mathbf{Y}_{l0}^{-}\} \exp\{-i[\pi/2(l+1)]\}$$

$$\mathbf{U}_2 = \sum_{l\geq 0} \{[a_l^{(2)} h_{l+1}(\omega\alpha_2 r) + l b_l^{(2)} h_{l+1}(\omega\beta_2 r)]\mathbf{Y}_{l0}^{+} + [-a_l^{(2)} h_{l-1}(\omega\alpha_2 r) + (l+1) b_l^{(2)} h_{l-1}(\omega\beta_2 r)]\mathbf{Y}_{l0}^{-}\} \exp\{-i[\pi/2(l+1)]\}$$

TASK: for each l find right coefficients a_l^i and b_l^i to match the boundary conditions at $r = R$

⇒ set of linear equations for each l

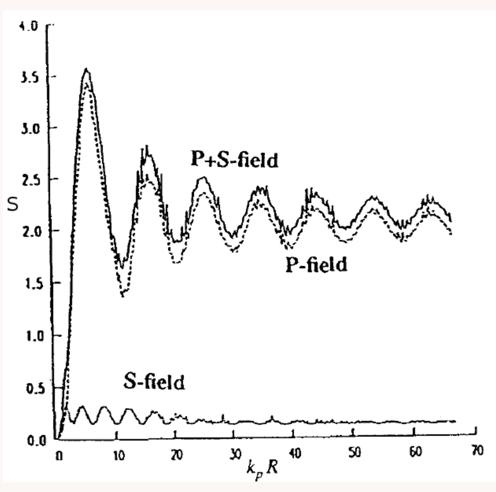
(Valeri Korneev, Geophys. J. Int. (1993))

Objectives from analytical approach

Scattering cross sections:
Ratio of scattered and incoming energy in dependence of frequency

$$s = \frac{F_{sc}}{F_0} \qquad F_0 = (\lambda + 2\mu)k\frac{\omega}{2} \qquad F_{sc} = \frac{\omega}{2}\mathrm{Im}\int_{Sphere\ r>R} (\mathrm{U}_2)\cdot \mathbf{t}\ ds$$

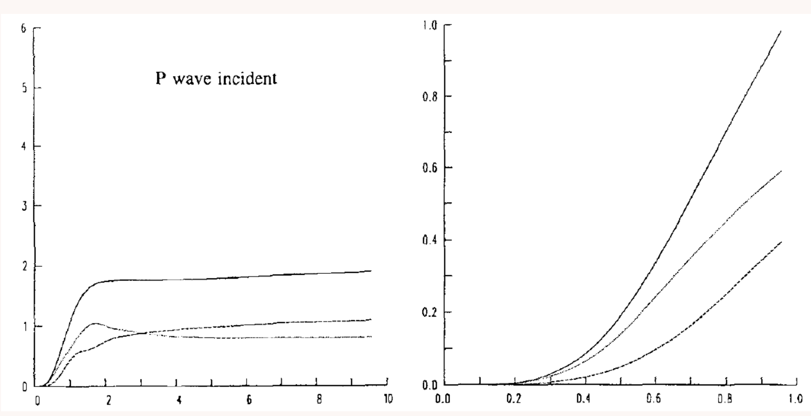
(incident P-wave) (scattered field)



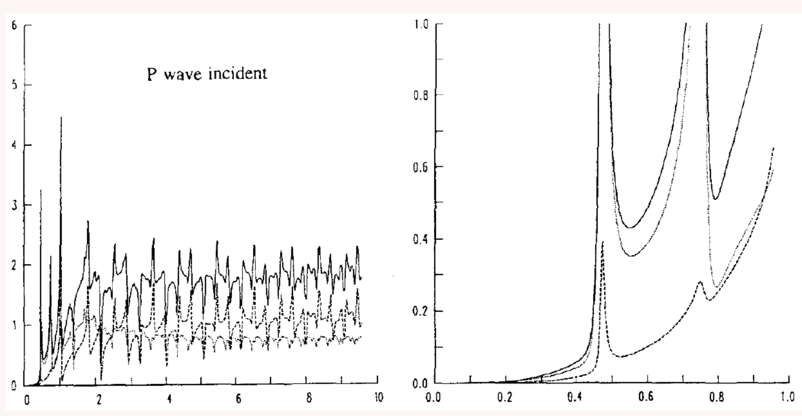
Elastic low velocity inclusion

- Large amplitude variation: interference of waves running through the inclusion and those running around it
- Small amplitude variation: multiple reflection within the cavity

Vacuum inclusion



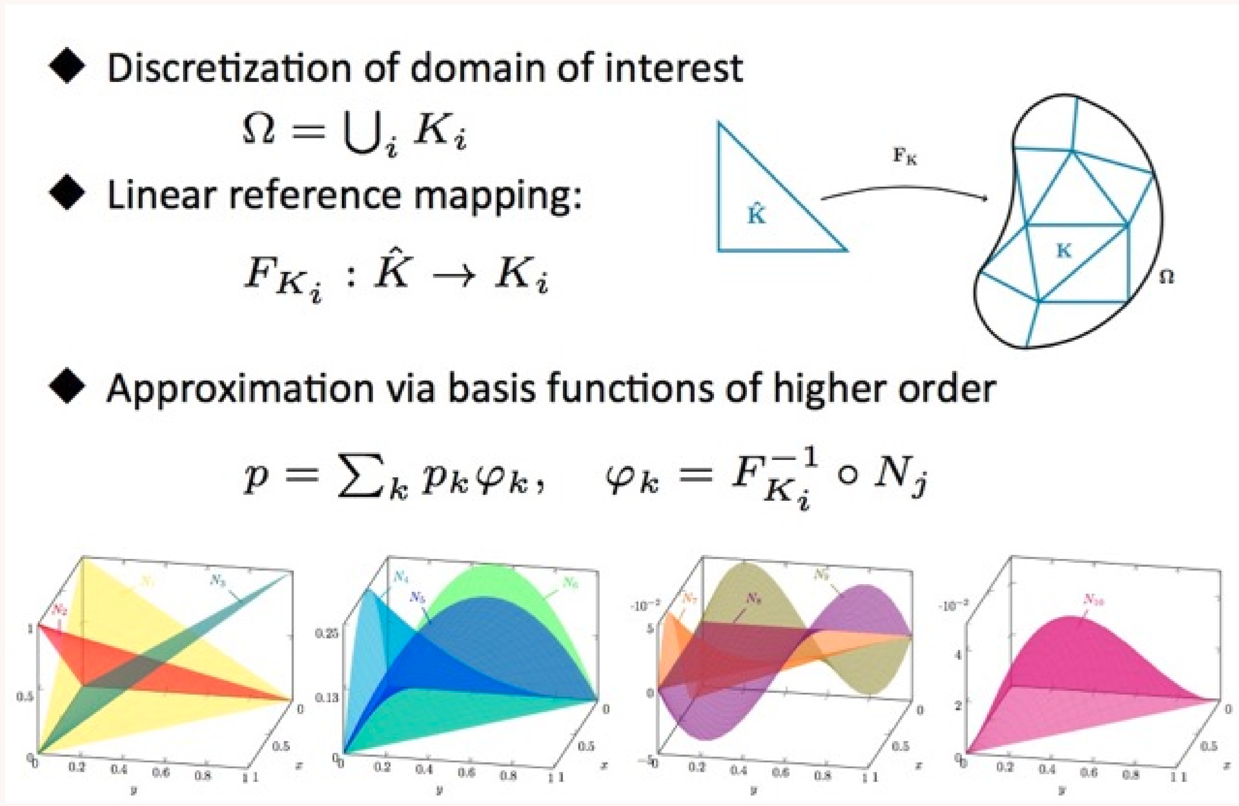
Acoustic inclusion



(Valeri Korneev, Geophys. J. Int. (1993))

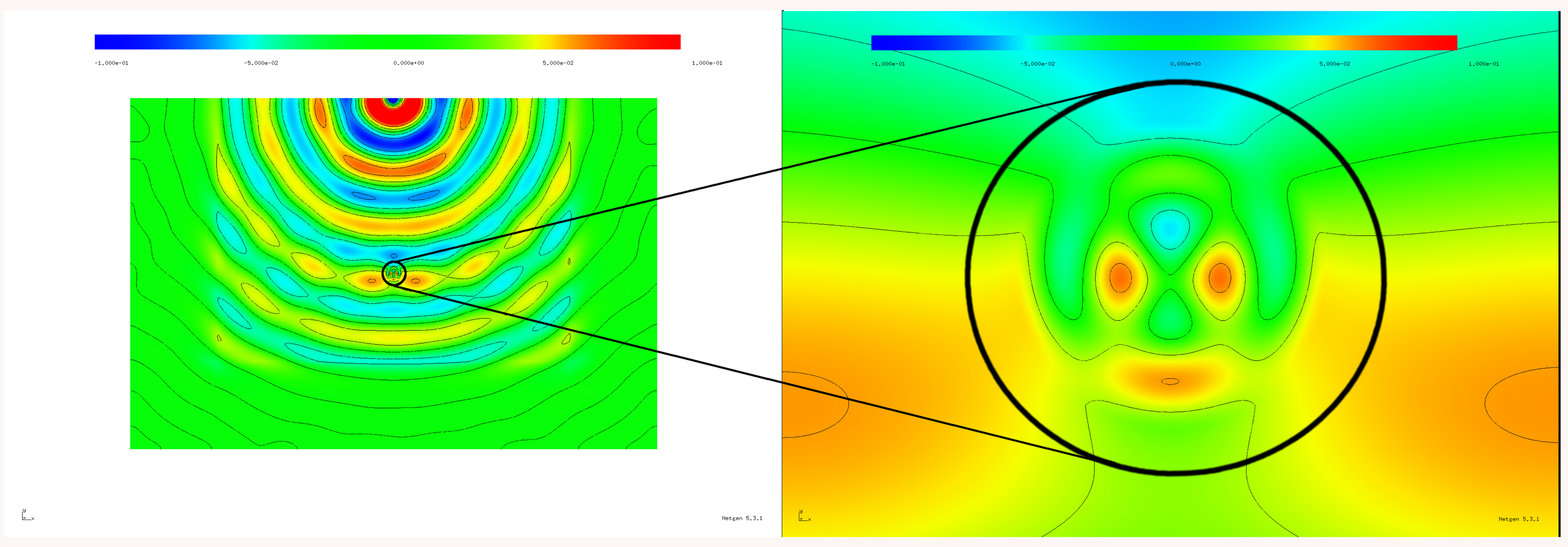
Numerical approach

Higher order finite element method:
NGSolve / Netgen (Joachim Schöberl, TU Vienna)

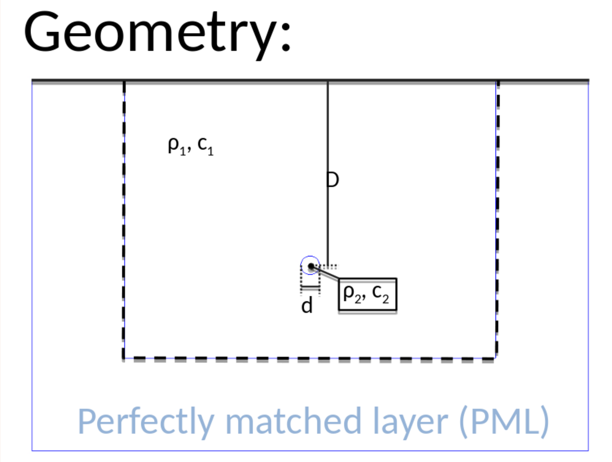


Forward modeling

- Total field for incoming point source at the surface above the cavity
- here: only acoustic case ($\mu = 0$ inside and outside of the cavity)



Geometry:



Physical parameters:

F = 16 Hz	
D = 600 m	d = 60 m
ρ_1 = 1000 kg/m³	c_1 = 3000 m/s
ρ_2 = 1 kg/m³	c_2 = 300 m/s

$\Rightarrow k_p R \approx 1$

Future target

- lay out scientific base for OSI techniques based on cavity wavefield interaction
  - → scattering
  - → resonance emission,...
- Design a method for cavity detection, either by artificial or natural sources